Transformations: Graphs of Equations

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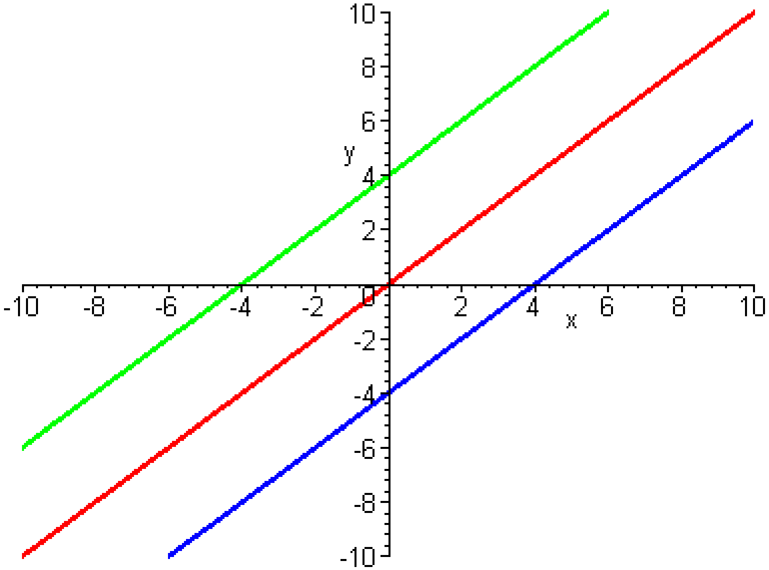
Algebra 1

Transformations: Graphs of Equations

- Graphical transformations (horizontal and/or vertical shifts) occur when coefficients (the numbers in front of the variables) and constants (the numbers after the variables) are changed.
- As you watch this presentation, try to identify the effect of changing the values of a, b, or c in the following types of equations:
  - Linear $y = ax + b$
  - Quadratic $y = a(x - b)^2 + c$
  - Absolute Value $y = a|x - b| + c$

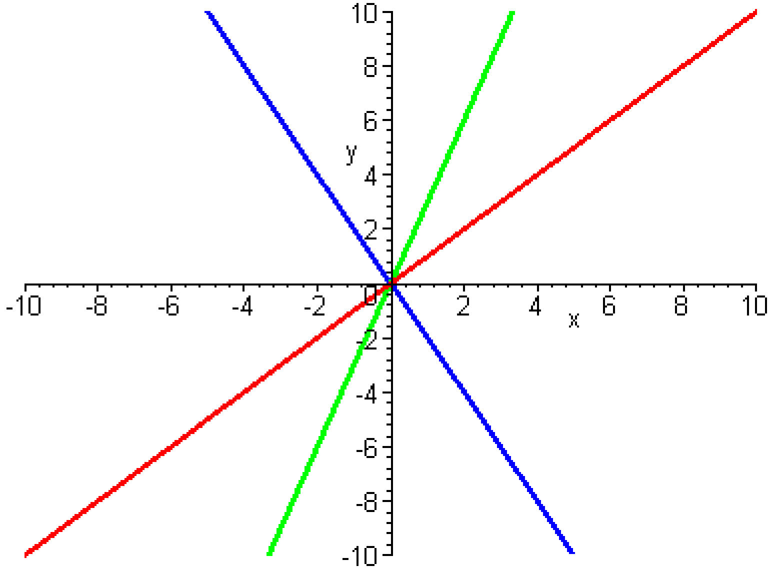
Transformations: Linear Equations

- The graph of a linear equation can be shifted vertically by changing the y-intercept.
- For example, $y = x$ is shifted up four units by changing the equation to $y = x + 4$ and down four units by changing the equation to $y = x - 4$.



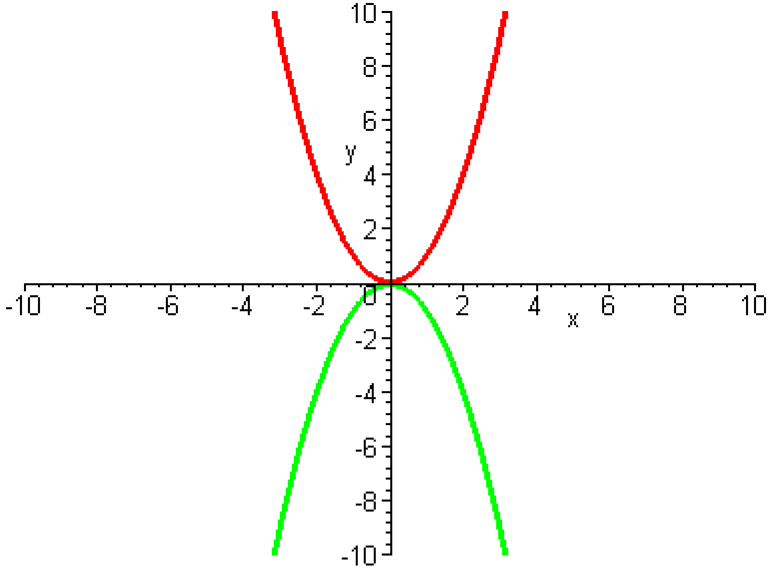
Transformations: Linear Equations

- The slope of a linear equation can be changed by increasing or decreasing the coefficient a in front of x.
- For example, y = ax
  - when a is positive (a > 0) the slope is positive and the slope increases as a increases: y = x, y = 3x
  - when a is negative (a < 0) the slope decreases as the absolute value of a increases: y = -2x



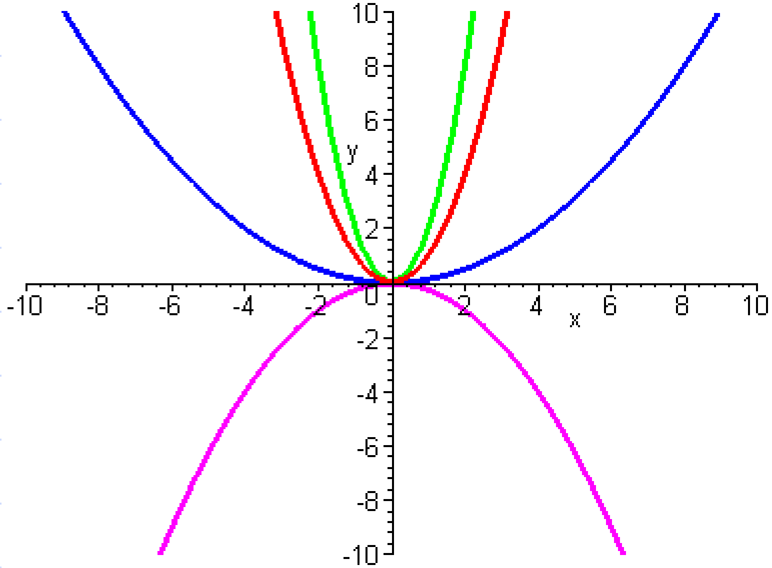
Transformations: Quadratic Equations

- In the standard graph of a quadratic equation, $y = ax^2$, the parabola opens upward when a is positive.
- When a is negative it opens downward and the graph is reflected (flipped upside down) about the x-axis: $y = -x^2$.



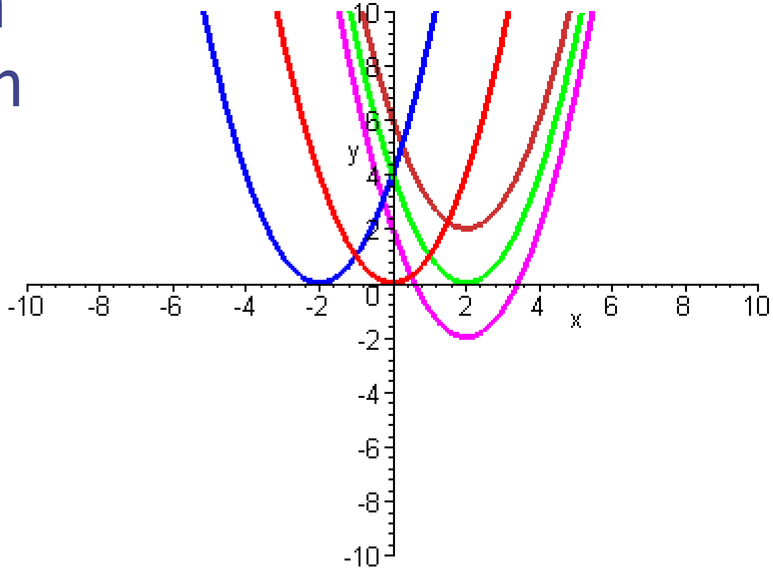
Transformations: Quadratic Equations

- In the standard graph of a quadratic equation $y = ax^2$, the opening of the parabola
  - narrows when the value of a increases: $y = x^2$, $y = 2x^2$
  - and widens as a decreases: $y = 1/8\ x^2$
- The graph is reflected about the x-axis when a is negative: $y = -\frac{1}{4}x^2$



Transformations: Quadratic Equations

Displaying the quadratic equation in the "complete the square" form of $y = a(x - b)^2 + c$ makes it easy to recognize:

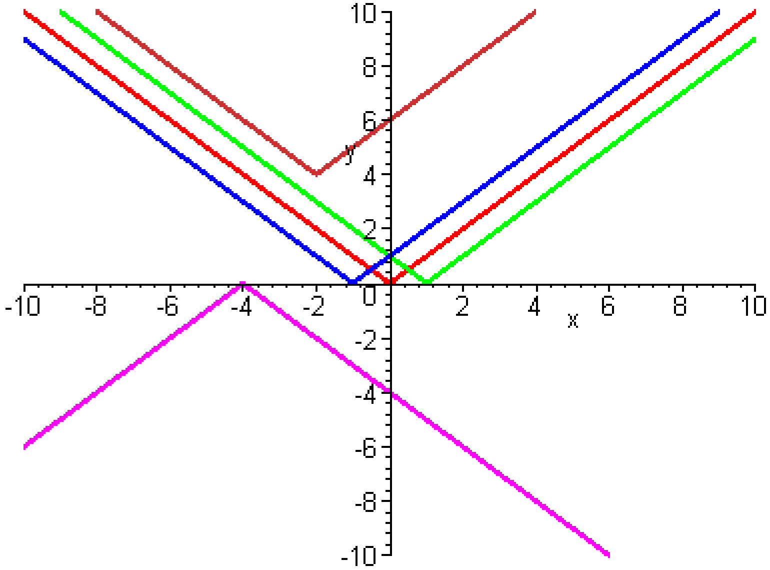


- horizontal shifts as b is changed:
  - $y = (x - 2)^2$ moves the graph of $y = x^2$ two spaces to the right,
  - and $y = (x + 2)^2$ moves it two spaces to the left.
- and vertical shifts as c is changed:
  - $y = (x - 2)^2 + 2$ shifts the graph right two spaces and up two spaces,
  - and $y = (x - 2)^2 - 2$ shifts the graph right two spaces and down two spaces.

Transformations: Absolute Value Equations

- In a similar manner, changes in a, b, and c of absolute value equations change the graph of $y = |x|$.

- Notice the changes in $y = |x|$ in the following graphs:
  - $y = |x - 1|$, $y = |x + 1|$
  - $y = |x + 2| + 4$, $y = -|x + 4|$



Transformations: Conclusions

- In general, what can you conclude about the effect of the coefficient a in front of the independent variable, or a in front of the parenthesis containing the independent variable, or a in front of the absolute value sign containing the independent variable?
- What can you conclude about the effect of the constant b on horizontal shifts?
- What can you conclude about the effect of the constant c on vertical shifts?
- Which way would you predict the graph of $y = x^3$ to shift when changed to $y = (x - 4)^3 + 2$? Use your graphing calculator to check your answer.

Linear	$y = ax + b$
Quadratic	$y = a(x - b)^2 + c$
Absolute Value	$y = a\|x - b\| + c$